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13. SUPPLEMENTARY NOTES

14. ABSTRACT

(1) Numerical analysis of a linearized formulation for three-dimensional buckling and stress-stiffening phenomena showed that an algorithm implemented in the commercial finite element code STRESS CHECK is robust. (2) An hp method was formulated and implemented for computing the response of woven composite materials. (3) Domain decomposition/concatenation hp methods were analyzed. (4) Locking-free robust hp mixed methods for elasticity were developed.

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15. SUBJECT TERMS

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2. Objectives

The following were the principal objectives of this project.

1. Numerical analysis of a linearized formulation for three-dimensional buckling and stress-stiffening phenomena. This work complemented an implementation on the commercial program STRESS CHECK.

2. Investigation of *hp* methods for computations over woven composite materials.

3. Modeling of multistructures via *hp* interface mixed methods for domain decomposition and concatenation. This work was originally begun to complement an implementation in the commercial program MSC-NASTRAN.

4. Development of *hp* methods that are robust and locking-free for elasticity problems.

3. Summary of Accomplishments/Status of Effort

1. The investigation of robustness for *hp* methods for the linearized buckling formulation implemented in STRESS CHECK was **completed**. It was found that the method was robust except in two main cases: (a) when the domain is not 'thin' and (b) when over-refinement is used in the presence of singularities. It was established that otherwise, one can be assured of accurate answers. The results were communicated to ESRD, the formulators of STRESS CHECK.

2. A specialized element for the study of woven composite materials, which incorporates various aspects of the structure of the material was formulated and tested. It gave excellent results in predicting the macro- and micromechanical response of the material.

3. The search for a non-conforming *hp* method that can be used for domain decomposition and concatenation was **completed** and several methods formulated and analyzed. The results were communicated to MacNeal-Schwendler, the formulators of MSC-NASTRAN.

4. A locking-free *hp* mixed formulation that is robust for elasticity problems even in the presence of curved boundaries was formulated and analyzed.

4. Accomplishments

Numerical analysis of a buckling model

The area of failure prediction, in the context of e.g. electronic components, ceramics, laminated composites, etc, has been of long-standing importance to USAF needs. Experimental determination of loads that can cause buckling and other failues is both expensive and unreliable. Mathematical models used in current engineering practice are, on the other hand, based on dimensionally reduced descriptions of an elastic body. The assumptions necessary for such dimensional reduction to hold can often result in a large, unknown modeling error when non-zero initial stress states are present, leading to inaccurate failure predictions.

A linearized model for buckling and stress-stiffening has been implemented in the *hp* code STRESSCHECK. This model does buckling analysis for the fully three-dimensional problem at hand, rather than some asymptotic (dimensionally reduced) limit. It finds the smallest positive multiple λ of an existing (pre-buckling) stress state that will result in buckling. The use of the *hp* method enables solutions over singular domains to be well approximated, and ensures that no locking takes place even when the domain is very thin.

However, a potentially serious danger of the method is that it characterizes λ as the lowest positive spectral value of a *non-compact* operator T. Such non-compact spectral value problems can be notoriously ill-behaved, due to the presence of spurious approximate eigenvalues, which can completely pollute the results.

The results we have obtained in this project establish that (1) spurious eigenvalues are *absent* for problems of engineering interest for which the pre-buckling stress σ_0 is bounded (2) spurious eigenvalues are *always present* for problems where σ_0 is unbounded. For the latter case, we demonstrate how the reliability of the computations can still be assessed, using the eigenvectors (the buckling shapes). We describe our results below.

1. Absence of spurious eigenvalues for bounded σ_0

Suppose Σ_B denotes the Browder spectrum of T, i.e. the set of all points in the spectrum which are not isolated eigenvalues of finite multiplicity. Then polluting eigenvalues only appear in the interval $[\min \Sigma_B, \max \Sigma_B]$. By considering the asymptotics of a plate of thickness d , we have shown that the Browder spectrum lies in an $O(d^2)$ interval around the origin, so that spectral pollution will not occur for the eigenvalues of interest, provided the domain is thin. (The thin case is the case of engineering interest.) See [5,7].

Again considering the asymptotics of a plate, we have investigated (with Monique Dauge) the effect of various initial stress states. We have shown

that the only case where spurious eigenvalues could occur is the one where the initial stress is of pure bending type. This case is, however, uninteresting from the point of view of applications. For all initial stresses that occur in applications (these can be decomposed into a bending and a non-zero membrane stress), spectral pollution does not occur in the region of interest.

Our asymptotic analysis has also established that boundary layers occurring in the solution do not lead to adverse affects in the approximation of the desired eigenvalues.

Finally, we have characterized the initial stresses that lead to pollution-free eigenvalue approximations when parts of the domain are not thin.

2. **Reliability of computations for unbounded σ_0**

For domains with corners, or domains with a change in the type of boundary conditions, it is well-known that the stresses can have $r^{-\alpha}$ singularities. We show that in this case, the Browder Spectrum will no longer be $O(d^2)$, but $O(d^2 * M)$ where M is dependent on the maximum of the stresses. Hence, the Browder Spectrum may be large enough to absorb the required eigenvalue.

In applications, one is interested in the lowest eigenvalue of a family related to the spectrum of a limiting problem (the $d = 0$ case). We characterize the values of α for which this limiting spectrum is still well-defined, and well-behaved. As long as the refinement is not too severe, we have demonstrated that we can still recover the lowest eigenvalue.

In Figure 1, we show typical buckling shapes associated with non-spurious modes (Fig 1(a) and 1(b)) and spurious modes (Fig 1(c)), for a plate with a 5 degree wedge cut out of it. The solution has a strong singularity at the wedge-tip. It is seen that for spurious modes, the eigenfunction is almost constant over the whole domain, except for the region surrounding the point of singularity (see the detail of an 'inner' element in Fig 1(d)). Figure 2 shows the convergence of the spurious modes when an *hp* method is used with n layers of refinement at the singularity. We see that the spurious eigenvalues converge to 0 at $O(e^{-Cn})$, and are independent of d. The limiting eigenvalues (the ones of interest) are almost independent of n (they have already converged). These decrease at an $O(d^2)$ rate as d is decreased. Hence, they can be recovered as long as they are smaller than the spurious eigenvalues, as shown in Figure 2.

Two important facts that emerge from our investigation are: (1) The degree of localness of the eigenfunctions can be an effective tool in assessing the reliability of the computations (2) There is often an optimal level of layer refinement, over which spurious eigenvalues can arise.

The above algorithm plays an essential role in an overall *hp* strategy for numerical computations. Our results indicate that subject to the above limitations (which are not serious in practice), the method is robust for en-

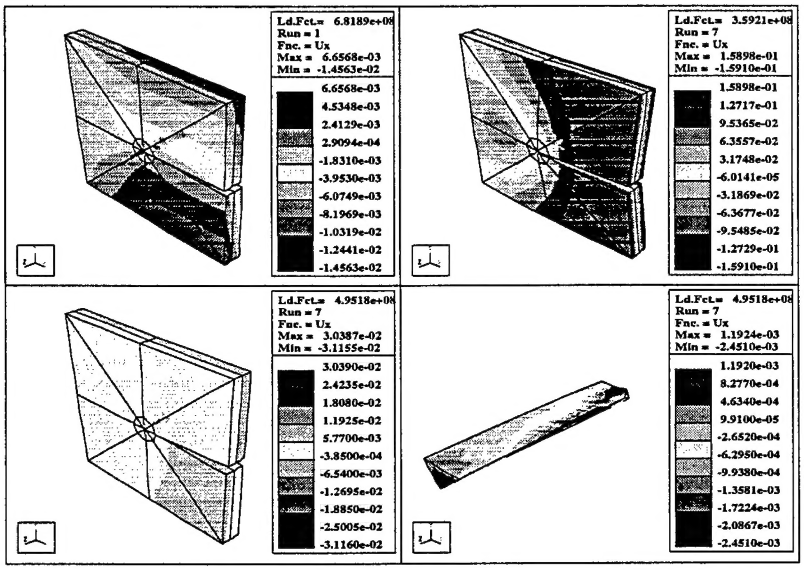

Figure 1: Buckling modes for thickness=0.1: (a) 1st mode, global (b) 2nd mode, global (c) 3rd mode, local (spurious) (d) Detail of 3rd mode

gineering applications.

Numerical methods for woven composites

The stated research objectives of AFOSR place significant emphasis on the use and performance prediction of highly heterogeneous composite media, capable of sustained use in adverse loading environments. These include, in particular, polymer matrix composites (PMCs) and ceramic matrix composites (CMCs). The structure of such materials is quite complex, with bundles of fibers (the *tows*) woven together and imbedded in a fully dense surrounding material (the *matrix*), with large voids usually present in CMCs. This complexity results in corresponding problems in their accurate numerical resolution. Previous computational efforts employ various modeling simplifications, with consequent repercussions in the resolution that can be achieved.

Our goal has been to develop an *hp* method for such materials. The starting point is to exploit the unit cell (Figure 3) which repeats throughout the material. This local element is modeled to incorporate the geometric and material features of the material (with care given to accurately modeling the geometry of the tows, for instance). Previous work by Prof. Charalambides of the Mech. Eng. Dept at UMBC has involved solving a series of canonical local boundary value problems on the local cell, to predict the response at this level. The local problems are first reduced to 2-d models by a Kirchhoff type

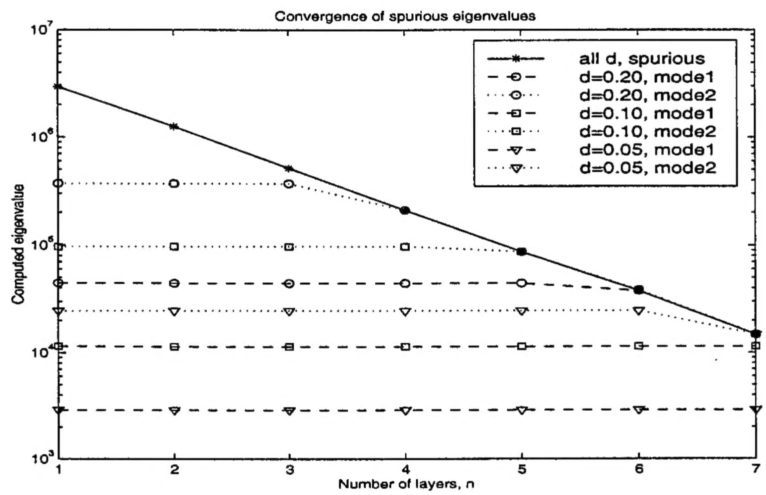

Figure 2: First, second and spurious e-values for various thicknesses d

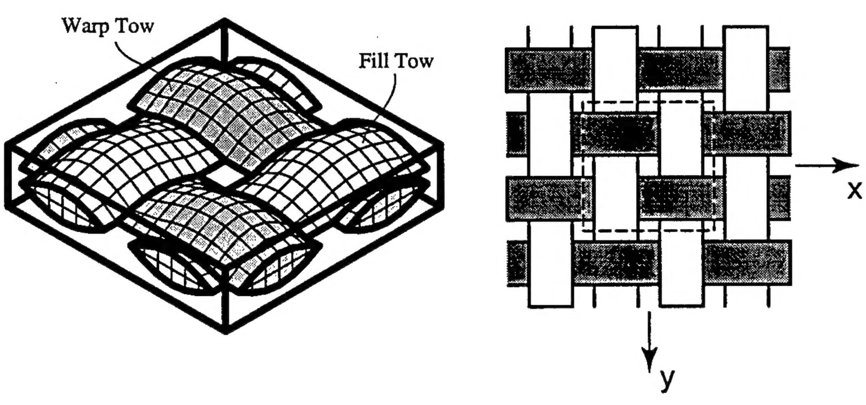

Figure 3: The plain weave composite repeating unit cell

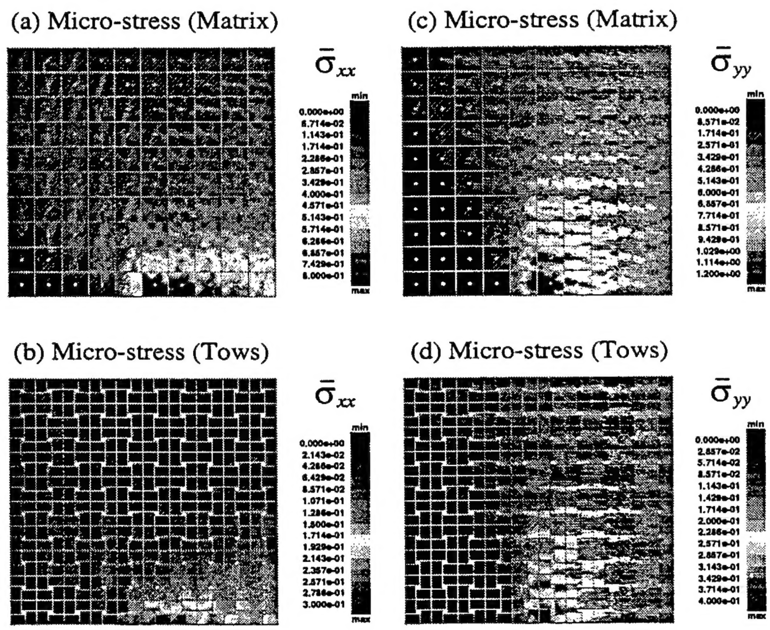

Figure 4: The micro-stress $\overline{\sigma}_{xx}$ of (a) the inter-tow matrix (b) the tows. The micro-stress $\overline{\sigma}_{yy}$ of (c) the inter-tow matrix (d) the tows.

hypothesis, and then solved by a p (spectral) method to avoid subdividing this cell. It has been empirically shown that the accuracy is comparable to that using a 20-noded brick element.

In joint work with Prof. Charalambides [6], we obtained the global response by assembling the local stiffness matrices and solving the resulting problem. Non-linear behavior is incorporated by updating the material properties selectively over the elements, based on the computed strains. The key factor is that mesh refinement never destroys the periodicity, since accuracy is increased only by increasing the polynomial degree. (The initial model assumes some simplifications, such as no provision for delamination or slipping of tows being included at this stage.)

We have formulated and implemented this specialized element and obtained computational results with it. In particular, we have compared these results to those obtained when calculations are performed using a homogenized version of the woven composite material. The macroscopic results match well when the domain has a large number of cells in it. The advantage with our approach, however, is that in addition to the macroscopic response, we are also able to obtain micro-strains and micro-stresses in various individual parts of the material. Such results are not possible when only homogenization is used.

Figure 4 demonstrates this. Here, we show the micro-stresses in both the matrix and the tow part of the composite. The problem is a standard near-tip crack problem, with the crack tip being at the center of the lower-most boundary of the square shown. The material is a CMC, which has a central circular void. It is clear that the inter-tow matrix phase and the tows phase exhibit different stress intensities, consistent with their intrinsic material dissimilarities. Moreover, in Figure 4(c) (showing the micro-stress $\overline{\sigma}_{yy}$ in the matrix), the stress concentration in the vicinity of the void hole at each unit-cell (represented by a narrow high-stress band perpendicular to its stress direction) is clearly captured. Currently available analysis tools that we are aware of generally use equivalent homogenized materials, and are unable to capture such details. (We note that such detail is needed for failure prediction at the micro level.)

Our future goal is to further develop this element by validating the assumptions that have been used in its design. Also, we will seek ways to improve its efficiency (since the calculation of the stiffness matrices currently requires a large amount of effort). We are also exploring methods to incorporate *hp* techniques of capturing crack-tip behavior.

Multistructure modeling via *hp* interface methods

Our previous work has shown how *hp* discretization of a complicated domain can be distributed over several independent meshers and then recomposed by mortaring techniques. In [1,2], we formulate new mortar methods that have optimal stability and approximability properties with respect to *hp* and *p*-discretization. Our results have been communicated to the makers of the *hp* code MSC-NASTRAN, where a similar method is implemented. The most-used application through this implementation is that "library" meshes on previously analyzed components are now being directly incorporated into global discretizations.

Robust *hp* mixed methods

We have continued our investigation of *hp* mixed methods, and shown how these can be applied to a problem of finite elasticity [3]. A survey of *hp* methods for domains like plates, shells, etc, has been prepared for a special JCAM issue on numerical methods [4].

5. Participating Personnel

In addition to the PI Manil Suri, the following people were associated with the research (but were not supported by the grant):
Collaborators: Barna Szabo (University of Washington, St. Louis, MO), Panos Charalambides and S.I. Haan (Dept of Mech. Eng, UMBC, MD), Ivo

Babuška (University of Texas, Austin, TX), Monique Dauge (University of Rennes, Rennes, France), Lawrence Chilton (AFIT, Wright-Patterson AFB, OH), F. Ben Belgacem (Univ. Paul Sabbatier, Toulose, France).
Ph.D. students: Padmanabhan Seshaiyar (Jul, 1998) and Alexandra Ward (current).

6. Publications

NOTE: Most publications, including recent preprints, can be retrieved from M. Suri's web page, http://www.math.umbc.edu/~suri.

1. P. Seshaiyer and M. Suri, "hp submeshing via non-conforming finite element methods," *Comp. Meth. Appl. Mech. Engrg.*, 189, 1011-1030 (2000).

2. Faker Ben Belgacem, Padmanabhan Seshaiyer and Manil Suri, "Optimal convergence rates of hp mortar finite element methods for second-order elliptic problems," *RAIRO Math. Mod. and Num. Anal.*, 34, 591-608 (2000).

3. Lawrence Chilton and Manil Suri, "Locking-free mixed hp finite element methods for curvilinear domains," *Comp. Meth. Appl. Mech. Engrg.*, 86, 29-48 (2000).

4. Manil Suri, "The p and hp finite element method for problems on thin domains," *J. Comp. and Appl. Math.*, 128, 235-260 (2001).

5. M. Suri and C. Xenophontos, "Reliability of an *hp* algorithm for buckling analysis," Proceedings of IASS-IACM 2000, Fourth International Colloquium on Computation of Shell and Spatial Structures, 2000.

6. S.I. Haan, P. Charalambides and Manil Suri, "A specialized finite element for the study of woven composites," *Computational Mechanics* (in press).

7. Manil Suri and Monique Dauge, "Numerical approximation of the spectra of non-compact operators arising in buckling problems." (preprint, 2001).

7. Interactions/Transitions

Presentations at meetings, conferences and seminars.

Mar '98 US Naval Academy, Annapolis, MD, "The *hp* finite element method over thin domains" (seminar)

Jun '98 "ICOSAHOM '98 (International Conference on Spectral and High Order Methods), Herzeliya, Israel, June 22-26, 1998, "Optimal approximation of singularities by non-conforming *hp* finite element methods" (invited minisymposium talk)

May '99 "SIAM Annual Meeting," Atlanta, GA, May 12-15, 1999, "Non-conforming *hp* finite elements: Mortaring techniques" (invited minisymposium talk)

Jul, '99 "ICIAM '99," Edinburgh, Scotland, Jul 5-9, 1999, "Optimal convergence rates of *hp* mortar finite element methods" (invited minisymposium talk)

Jul, '99 "ICIAM '99," Edinburgh, Scotland, Jul 5-9, 1999, "On the approximation of spectra for buckling problems" (invited minisymposium talk)

Oct, '99 "AMS Sectional Meeting," Austin, TX, Oct 8-10, 1999, "hp methods for buckling problems" (invited talk)

Apr, '00 "Elastic Shells: Modeling, Analysis and Numerics" MSRI, Berkeley, CA, Apr 17-28, 2000, "The numerical analysis of an *hp* algorithm for the approximation of buckling problems" (invited plenary talk)

May, '00 "*p* and *hp* Finite Element Methods: Mathematics and Engineering Practice" St. Louis, MO, May 31-June 2, 2000, **Organizer of Conference**, "Reliability of an *hp* algorithm for buckling analysis"

Jun, '00 "IASS - IACM 2000 Fourth International Colloquium on Computation of Shell and Spatial Structures" Chania, Crete, June 4-7, 2000, "Reliability of an *hp* algorithm for buckling analysis" (invited minisymposium talk)

Apr, '01 "Conference in Honor of Jim Greenberg's 60th Birthday" Carnegie-Mellon University, Pittsburgh, April 20-21, 2001, "The approximation of the spectra of non-compact operators arising in buckling analysis" (plenary talk)

May, '01 Colloquium talk, Dept of Mathematics, University of New South Wales, Sydney, Australia, May 16, 2001, "The approximation of the spectra of non-compact operators arising in buckling analysis"

May, '01 Colloquium talk, Tata Institute of Fundamental Research, Mumbai, India, May 31, 2001, "Problems in finite elements"

Jun, '01 "ICOSAHOM '01 (International Conference on Spectral and High Order Methods), Uppsala, Sweden, June 11-15, 2001, "Reliability of an *hp* finite element method for buckling analysis" (invited minisymposium talk)

Transitions

1. Organized conference on *p* and *hp* methods in St. Louis with support of AFOSR. Dominant thrust of the conference was the dissemination of AFOSR-supported (and other) research in *p*/*hp* finite elements by mathematicians and engineers to the industrial community.

2. Research on non-linear buckling is in collaboration with Prof. Barna Szabo of ESRD, 7750 Clayton Road, St. Louis, Missouri 63117 (phone: 314-645-1423). Technology transfer is occurring through the implementation of this method in the commercial code STRESS CHECK.

3. As reported in the last grant, work on non-conforming *hp* methods for multistructures helped provide theoretical validation for an interface method implemented in the commercial code MSC-NASTRAN. Contact: Dr. John Schiermeier, The MacNeal-Schwendler Corporation, 815 Colorado Blvd, Los Angeles, CA 90041-1777 (phone: 213-259-3832).

8. Inventions/patent disclosures: None.

9. Honors and Awards: Appointed to editorial board of SIAM Journal on Numerical Analysis.